Dynamics and Kinetics. Exercise 7

Problem 1

Starting from the Maxwell-Boltzmann distribution of velocity magnitudes,

$$f_{\mathrm{MB}}(v)dv = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right) 4\pi v^2 dv,$$

find the expressions for the root mean square velocity and for the most probable velocity.

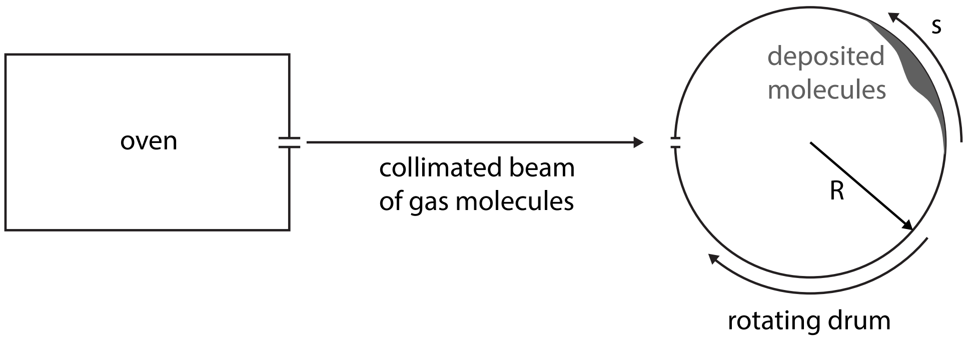
Problem 2

Starting from the Maxwell-Boltzmann distribution of velocity magnitudes (see Problem 1), derive the Maxwell-Boltzmann distribution of kinetic energies,

$$f(\varepsilon)d\varepsilon = 2\pi\left(\pi k_B T\right)^{-3/2} \varepsilon^{1/2} e^{-\varepsilon/k_B T} d\varepsilon.$$

Problem 3

The figure below illustrates an experiment for measuring the Maxwell-Boltzmann distribution.



An oven at temperature T releases a narrow beam of gas molecules of mass m through a hole. The molecules strike a drum of radius R that rotates with frequency ν. The drum has a small opening through which gas molecules can enter into the drum when the opening passes through the beam of gas molecules. Because of the fast rotation of the drum, only a short pulse of gas molecules enters. Once these molecules reach the opposite wall of the drum, they stick to it.

a) Show that the flux of molecules of velocity u contained in a pulse that enters the drum is proportional to

$$u^3 e^{-\frac{mu^2}{2k_B T}} du$$

b) The molecules are deposited on the wall of the drum at a distance s from the point opposite the opening in the drum as indicated in the figure. Derive the distribution $I(s)ds$ of the deposited molecules. It is not necessary to normalize this distribution.

Problem 4

Derive the speed distribution $F(u)du$ of a two-dimensional ideal gas.

Hint: Start from a one-dimensional velocity distribution to derive a two-dimensional distribution of the velocities, and then do a suitable coordinate transformation.